Probing dipolar effects with condensate shape oscillation

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We discuss the low energy shape oscillations of a magnetic trapped atomic condensate including the spin dipole interaction. When the nominal isotropic s-wave interaction strength becomes tunable through a Feshbach resonance (e.g. as for ^{85}Rb atoms), anisotropic dipolar effects are shown to be detectable under current experimental conditions [E. A. Donley *et al.*, Nature **412**, 295 (2001)].

Collective excitations play an important role in probing microscopic interactions [1]. The recently available gaseous atomic Bose-Einstein condensates (BEC), have proven to be a profitable testing ground for such studies [2]. Atomic BEC are dilute with properties completely determined by binary interactions. Experimentally they are created at very low temperatures when the short range atom-atom (collision) interaction can be described by a single parameter: the s-wave scattering length $a_{\rm sc}$. All higher order partial wave collisions are suppressed at zero collision energy in a short ranged potential. This enable atoms to be modelled as hard spheres of radius $a_{\rm sc}$, reflecting the isotropic ground state interaction. Inside an electric or magnetic field, however, ground state atoms may be polarized, e.g. in a magnetic trap the direction of atomic spins (of the valance electron for alkali) becomes aligned. The resulting dipole interaction between condensed atom pairs many not simply average out. In this article, we investigate such spin dipole effects on shape oscillation frequencies of an atomic condensate. Although dipolar effects are typically small compared to the dominant s-wave contact interaction, our study indicates that these shifts become observable in the ^{85}Rb BEC [3, 4], when a Feshbach resonance is used to tune $a_{\rm sc}$ near zero [3].

Dipolar interaction in atomic BEC leads to physics beyond the usual s-wave contact term, mainly because of the modified low energy collision threshold behavior due to the anisotropic nature of the interaction [5]. For magnetic spin dipoles, the net effect in the dilute gas sample is rather small, one can therefor approximate the complete two-body interaction by [6, 7]

$$V(\vec{R}) = g_0\delta(\vec{R}) + g_2\frac{1-3\cos^2\theta_R}{R^3} \tag{1}$$

where $\vec{R}=\vec{r}-\vec{r}\,'$, and $g_0=4\pi\hbar^2 a_{\rm sc}/M$ is the s-wave contact term. More generally, the dipole strength g_2 equals $\alpha^2(0)\mathcal{E}^2$ ($\alpha(0)$ atomic polarizability) or μ^2 (μ magnetic dipole moment) respectively for electric or magnetic dipoles. Figure 1 illustrates the geometry for two aligned dipoles along the local magnetic field (z-axis). Several interesting properties of a dipolar condensate have already been discussed [6, 7, 8, 9]. In this paper, we focus on the shape oscillations of a trapped dipolar condensate assuming a tunable $a_{\rm sc}$.

In the standard approach, low energy collective excitations are described by the Bogoliubov theory [10]. The condensate wavefunction $\phi(\vec{r},t)$ is governed by the Gross-Pitaevskii equation, while the non-condensed atoms are described by quasi-particles [11], some of which have been studied experimentally [12, 13]. The inclusion of the non-local dipolar interaction makes the Bogoliubov approach impractical to implement numerically. We therefore will rely on two alternative methods to study the three characteristic shape modes [12] as graphically illustrated for a cylindrical symmetric trap in Fig. 2.

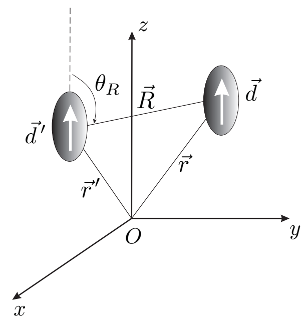

FIG. 1: Geometry for the interaction of two aligned dipoles.

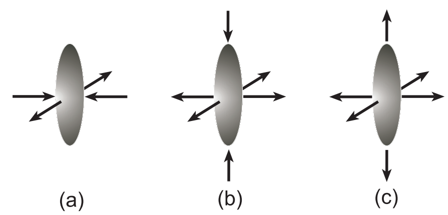

FIG. 2: Collective excitations of a condensate with cylindrical symmetry. Mode a is purely radial due to the cylindrical symmetry. It's angular momentum projection along the z-axis $m=2$. Mode b and c are respectively the quadrupole and monopole oscillations. They are customarily called the low and high $m=0$ modes.

First we adopt the highly successful time-dependent variation approach used in Ref. [14] by assuming a Gaussian ansatz

$$\phi(x,y,z,t) = A(t)\prod_{\eta=x,y,z} e^{-\eta^2/2q_\eta^2+i\eta^2\beta_\eta(t)}. \tag{2}$$

For an harmonic trap $V_t = M\sum_{\eta=x,y,z}\nu_\eta^2\eta^2/2$ ($\nu_\eta=\omega\lambda_\eta$), the equations for variational parameters q_η are equivalent to the classical motion of a particle (with co-

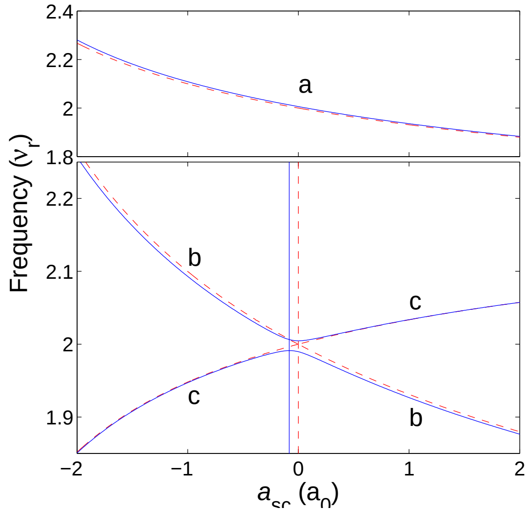

FIG. 3: Mode frequencies for $N = 10^4$ and $\lambda = 1$. Solid (dashed) lines are results excluding (including) dipole interactions. a_0 stands for Bohr radius.

ordinate q_η) inside an effective potential

$$\begin{aligned} &U(q_x, q_y, q_z) \\ &= \sum_{\eta=x,y,z} \left(\frac{\hbar^2}{2Mq_\eta^2} + \frac{M\omega^2}{2}\lambda_\eta^2 q_\eta^2 \right) + \frac{Ng_0}{(2\pi)^{3/2} q_x q_y q_z} \\ &+ \ N\frac{g_2}{(2\pi)^{3/2}} \frac{1}{q_x q_y q_z} \int d\vec{r}\, \frac{1-3\cos^2\theta}{r^3}\, e^{-\sum_\eta \eta^2/2q_\eta^2}, \end{aligned} \quad (3)$$

where N is the number of condensate atoms. The equilibrium location (q_η^0) of Eq. (3) then determines condensate size, while linearized shape oscillation frequencies are determined by the second order derivative $U_{\eta\eta'}(q_x, q_y, q_z)$ evaluated at q_η^0, For a cylindrically symmetric trap ($\lambda_x = \lambda_y = 1$), we take $q_x = q_y = q_r$ and $\lambda_z = \lambda$. The last integral in (3) as well as $U_{\eta\eta'}$ all become analytically computable [9]. The resulting $U_{\eta\eta'}$ matrix is symmetric ($U_{\eta\eta'} = U_{\eta'\eta'}$), diagonalization of which gives the three mode frequencies $\nu_a = \sqrt{U_{11} - U_{12}}$ and

$$\nu_{b,c} = \frac{1}{\sqrt{2}} \Big[\quad U_{11} + U_{12} + U_{33} \\ \pm\sqrt{(U_{11}^2 + U_{12}^2 - U_{33}^2)^2 + 8U_{13}^2} \Big]^{1/2}. \quad (4)$$

In the JILA experiments [3, 4] with ^{85}Rb in the $|F = 2, M_F = 2\rangle$ state, the valance electron spin gives rise to an aligned magnetic dipole moment of $\mu = 2\mu_B/3$ (μ_B being the Bohr magneton). We take the radial frequency to be $\nu_r = 17.35$ (Hz), in addition to a tunable $a_{\rm sc}$ and trap aspect ratio λ as in the experiment [4]. Our results confirm that spin dipole effects become detectable in terms of shifts of the shape oscillation frequencies. We first report variational calculated results as the analytic formulae obtained allow for a careful analysis of the underline physics.

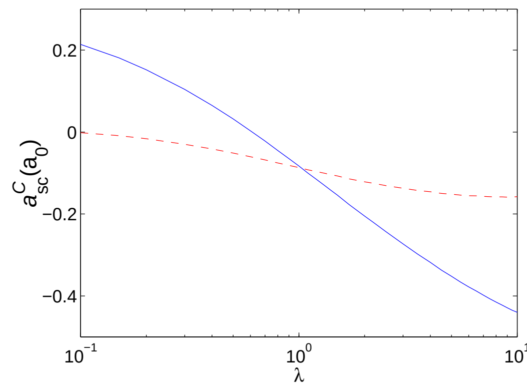

FIG. 4: The λ dependence of the critical scattering length $a_{\rm sc}^C$ for $N = 10^4$. The overall trend agrees with the variational result for $a_{\rm eff}$ (in dashed line) derived earlier in Ref. [8].

Figure 3 shows the three mode frequencies for $N = 10^4$ in a spherical trap ($\lambda = 1$). The solid and dashed lines denote respectively mode frequencies without and with the dipolar interaction. The vertical lines show the critical values of $a_{\rm sc}^C$ when the mode character switching $b \leftrightarrow c$ occurs. This mode switching occurs whenever the overall mean-field condensate interaction changes from repulsive to attractive or *vice versa*. When squeezed along the radial direction, mode c (b) became predominantly excited for an overall attractive (repulsive) condensate as atoms are pulled in (pushed out) along the z-direction. Without the spin dipole interaction, this mode switch always occurs at $a_{\rm sc} = 0$. The anisotropic dipole interaction affects the overall condensate mean field and the stability depending on the trap aspect ratio λ [6, 8]. Depending on the configuration of the dipoles, dipole-dipole interactions can be either attractive or repulsive. For two dipoles, if they were placed in a plane perpendicular to their polarization ($\uparrow\uparrow$), they repel each other. On the other hand, they attract each other if they were placed along the direction of their polarization ($\rightarrow\rightarrow$). For a cloud of trapped dipoles one therefore expects that repulsive interaction increases as one increases λ, which leads to increased condensate stability. In Fig. 4, we display the λ dependence of the critical scattering length $a_{\rm sc}^C$ as obtained from the variational calculation. We find that the effective scattering length $a_{\rm eff}$ (introduced earlier by us in [6]) provides a reasonable approximation to the sign of the overall condensate mean field. In terms of the actual values, however, the results in the figure show that $a_{\rm eff}$ differs significantly from $a_{\rm sc}^C$ when λ deviates from the neighborhood of unity.

In Fig. 5, we show the λ-dependence of the dipole induced fractional changes to the mode frequencies for a condensate with 10^4 atoms at $a_{\rm sc} = -1\,(a_0)$. For strongly prolate or oblate traps, the shifts are in a few percentage range even with such small numbers of atom. Figure 6 summarizes our results for the atom number dependence of the dipole induced frequency shifts for the trap aspect ratio $\lambda = 6.8/17.35$ [4]. It was shown earlier [9], at

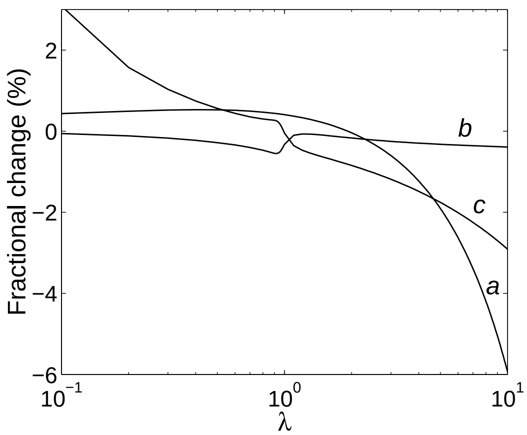

FIG. 5: Fractional change (%) of mode frequencies for $a_{\rm sc} = -1\,(a_0)$ and $N = 10^4$. Similar results are obtained for $a_{\rm sc} = 0$ and $1\,(a_0)$.

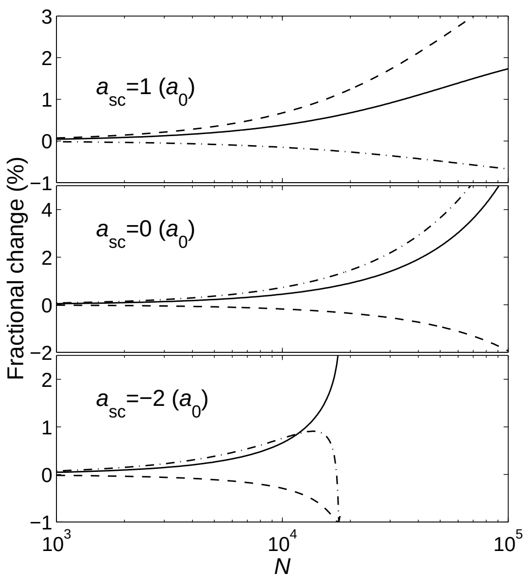

FIG. 6: Atom number dependence of the dipole interaction induced shape oscillation frequency shifts. Mode a, b, and c are labelled respective with solid, dashed, and dot-dashed lines.

increased values of N, a dipolar condensate always collapses irrespective of the sign of $a_{\rm sc}$. The Gaussian ansatz (2) becomes questionable near collapse as reflected in the seemly divergent results when N is increased.

These results (Figs. 3-6) clearly show that dipolar effects are detectable in the current ^{85}Rb BEC setup, given the extraordinary capability of 0.1% frequency measurement [16]. To confirm the validity of the variational calculations, we have invested considerable effort in an exact numerical method based on the time dependent Gross-Pitaevskii equation [17]. By applying external periodic

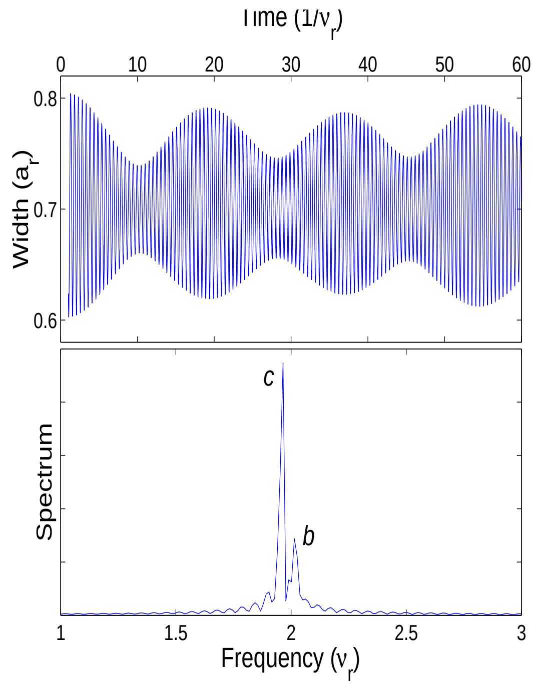

FIG. 7: Typical numerical results for condensate width and its corresponding Fouier transformed signal. Mode b and c are clearly identified. The parameters are $a_{\rm sc} = 0$, $\lambda = 1$, and $N = 40000$.

forcing terms described by the potential

$$V_F(\vec{r}) = \sum_{\eta=x,y,z} V_\eta \left\{ e^{-\frac{[\eta+\Delta_\eta(t)]^2}{2w^2}} + e^{-\frac{[\eta-\Delta_\eta(t)]^2}{2w^2}} \right\},$$

with $\Delta_\eta(t) = \eta_0 + \delta_\eta \sin(\Omega_\eta t + \phi_\eta)$ to the condensate ground state, selected shape oscillations become predominantly excited, as first numerically implemented by Ruprecht *et al.* [17].

After a selected duration T, typically several periods of the trap radial oscillation, the forcing term V_F is tuned off. The free propagation of the time dependent Gross-Pitaevskii equation is continued, and the dynamic condensate width $\sqrt{\langle \eta^2(t) \rangle}$ is sampled. The shape oscillation frequencies are then identified by taking the Fourier transformation of $\sqrt{\langle \eta^2(t) \rangle}$. A typical result from this calculation is given in Fig. 7, which shows remarkably clear signal. By varying V_η, η_0, δ_η, and Ω_η, our results are self-consistently checked, i.e. to be independent of all parameters involved as they should be in the small amplitude oscillation limit and to be numerically accurate. For the experimental parameter of $\nu_r = 17.35$ (Hz) and $\nu_z = 6.8$ (Hz). We have computed the shape oscillation frequencies numerically for $a_{\rm sc} = 0, 1, -2\ (a_0)$, and compared them with the variational results in Table I. The quoted errors for the numerical results shown in the tables mainly come from the limited sampling window in time. Further improvement is hard as the calculations are time consuming and numerical accuracies become difficult to control at long times. We find that

TABLE I: Mode frequencies (in unit of ν_r) with dipolar interaction

$a_{\rm sc}(a_0)$	mode	numerical	variational
-2	b	2.0010 ± 0.0040	2.0016
	c	0.8578 ± 0.0085	0.8657
0	b	1.9959 ± 0.0028	1.9964
	c	0.7874 ± 0.0022	0.7895
$+1$	b	0.7705 ± 0.0060	0.7675
	c	1.9974 ± 0.0032	1.9974

TABLE II: Mode frequencies (in unit of ν_r) without dipolar interaction

$a_{\rm sc}(a_0)$	mode	numerical	variational
-2	b	2.0073 ± 0.0025	2.0075
	c	0.8546 ± 0.0051	0.8591
$+1$	b	0.7664 ± 0.0054	0.7622
	c	2.0004 ± 0.0024	2.0004

the variational results are consistent with the exact numerical results. In order to affirm such mode frequency shifts are indeed from the dipolar interaction, rather than a mis-calibration of $a_{\rm sc}$, we also need to validate the variational approach in the absence of the dipole interaction. This was explored earlier in Ref. [14], where the numerical and variational results of the condensate dynamics were compared. Our results are presented in Table II. It convincingly proves that dipolar efforts reported in the Table I is due to the physics of spin dipole interaction.

In conclusion, we have studied low energy shape oscillations of a trapped dipolar condensate. Using a magnetic field dependent Feshbach resonance to tune the s-wave scattering length to around zero, we have shown that the weak spin dipole interaction becomes detectable as shifts to shape oscillation frequencies under currently available experimental conditions [3, 4]. These shifts grow with the number of trapped atoms, and are around 1% level with as few as 10^4 atoms. We have also independently verified the accuracy of the variational calculation by developing a rigorous numerical approach for the three predominant shape oscillation modes. Near a Feshbach resonance, significant atom loss might occur as found in the Na experiment [18]. The subsequent damping could lead to a broadening of the shape oscillation thus masking the direct observation of the proposed spin dipole effects. Fortunately for ^{85}Rb atoms, the condition of $a_{\rm sc}(B) = 0$ corresponds to far off resonance on the high B-field side, where impressive controls have been demonstrated without any significant loss [4]. Over the last few years, mean field theory has proven to be remarkably successful when applied to BEC physics. The s-wave g_0 contact pseudo-potential has made the concept of scattering length $a_{\rm sc}$ widely popular. It is often said that scattering length is the only relevant atomic parameter since the net interaction effect scales as $Na_{\rm sc}/a_{\rm ho}$ for a harmonically trapped atomic condensate, where $a_{\rm ho}$ is the trap size. Dipole interactions as discussed in this article points to interesting physics beyond the s-wave. Within the current experimental regime of dilute atomic gases, mean field theory remains applicable and allows for the calculation of dipolar induced shifts of collective excitation frequencies. Successful experimental verification of our predictions will shed new light on atomic BEC.

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